# Choice waves and strategic interaction

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## ABSTRACT

The Choice Wave provides a conceptual and mathematical framework for probabilistic choice leading to one or more utility maximizing outcomes chosen independently at various decision points. Yet, there exists the possibility that individuals may interact strategically with other individuals that may be represented by Choice Waves different from their own. When the potential for strategic interaction between individuals in one Choice Type and individuals in another Choice Type exists, there exists the logical possibility that the probabilities of utility maximizing outcomes may change. Because the players exist in separate hyperspace, if their outcome is influenced by strategic interaction, then their very interaction in the game, the game itself becomes a new entity with its own Choice Wave that is a combination of some sort of the Choice Waves of each player. The game itself becomes the entity that is making the final decision, which does not take place until each player makes a decision, each of which is conditional on the decisions of other players. Since Transactional Analysis games provide good examples of human subconscious strategic interaction that often ends in a Nash equilibrium, this study considers strategic interaction between two players of a specific TA game known as "Now I've Got You." The Nash equilibrium outcome is modelled according to Choice Waves, which is then used to create a probabilistic demand model that incorporates the possibility of outcomes that are affected by transactional analysis games.

Keywords: Choice Wave, Nash Equilibrium, Strategic Interaction, Probabilistic Demand, Transactional Analysis

## INTRODUCTION

The Choice Wave provides a conceptual and mathematical framework for probabilistic choice leading to one or more utility maximizing outcomes chosen independently at various decision points. Individuals or groups of individuals (known as Consumer Types in market settings, or otherwise as Choice Types) that can successfully be represented by a Choice Wave are statistically independent from all other consumers or Choice Types represented by different Choice Waves due to the orthogonality of each Choice Wave (Johnson, 2011). Yet, there exists the possibility that individuals may interact strategically with other individuals that may be represented by Choice Waves different from their own (Johnson, 2013, 2015). Given that a Choice Wave represents the complete set of utility maximizing choices, each of which has a certain probability that the individual will select that choice at the decision point, there necessarily is an expectation value associated with each Choice Wave that indicates the most likely outcome or outcomes. Two or more outcomes may certainly have equivalent likelihood of occurrence, depending on the tastes and preferences and other factors, which are included in the form of the Choice Wave. In general, each Choice Type exists within its own hyperspace such that the behavior and preferences of individuals in one Choice Type are statistically different from those of all other Choice Types. However, when the potential for strategic interaction between individuals in one Choice Type and individuals in another Choice Type exists, there exists the logical possibility that the probabilities of utility maximizing outcomes may change. Different Choice Types may respond differently to being confronted with strategic interactions much in the same way that they may respond differently to information (Teisl, Roe, and Hicks, 2002; Johnson, 2016). However, information, once provided, is static, unlike individuals, who have the capacity to act in accordance with their own utility maximizing strategies. Thus, the utility maximization problem at a decision point in the presence of a strategic interaction situation is not merely related to the individual's own Choice Wave, but is also conditional on the Choice Waves of other individuals with which the interaction is taking place. This poses an interesting question, since two Choice Types are supposed to be different in terms of behavior and preferences, and therefore not influence each other's decisions. Such differences, however, logically do not apply to strategic interaction – that is, there may exist a possibility for certain types of interaction between individuals that otherwise exist orthogonally to each other. Even the most different of individuals may interact strategically. For example, nations with diametrically opposed political and social views that could be represented by different Choice Waves nevertheless interact and may become involved in a war despite being otherwise statistically independent in terms of behavior and preferences (Bausch, 2015). The scenario is one of Nash Equilibria, in which strategic interaction between two or more players yields a dominant or multiple strategies for the players. This can be highly dependent on the amount and type of information each player has regarding the intentions and strategies of the other player or players. More information yields better results, and that information may come through a series of signaling processes as an interaction unfolds (Cobb, Basuchoudhary, and Hartman, 2013).

The pre-interaction expectation values of the Choice Waves of the players may provide additional insight into the most likely outcome of the game once strategic interaction takes place and the expectation value is conditional upon the Choice Waves of the other players, whether it is a game with a dominant strategy or multiple strategies. Additionally, because the players exist in separate hyperspace, if their outcome is influenced by strategic interaction, then their very interaction in the game, the game itself becomes a new entity with its own Choice Wave that is a combination of some type of the Choice Waves of each player. The game itself becomes the entity that is making the final decision, which does not take place until each player makes a decision, each of which is conditional on the decisions of other players. Therefore, during the period of strategic interaction, the players exist within a new hyperspace that is shared by all players. For example, two sports teams, A & B, playing each other have their own possible outcomes, i.e., win, lose, or tie, each of which is dependent upon the outcome of the other player. The game itself has its own possible outcomes, i.e., A wins, B wins, A and B tie. The outcome of each team is contingent upon the outcome of each team, and the outcome of each of the teams is contingent upon the outcome of the game. Insofar as the utility maximizing decision possibilities of each team may legitimately be modeled by a Choice Wave orthogonal to the Choice Wave of the other team, then the game itself may be modeled by a Choice Wave that is a combination of some type of the Choice Waves of the two teams, thereby placing the two teams, during the period of the game, on a new hyperspace that they both share. The game may be thought of as making a collective choice that yields a certain outcome, which is a combination of some type of the outcomes of each team.

Transactional Analysis games provide good examples of human strategic interaction that often ends in a Nash equilibrium. Transactional Analysis (TA) was established as a branch of behavioral science in the 1960s by Eric Berne, an American medical doctor. This study considers strategic interaction between two players of a specific TA game known as “Now I’ve Got You” that has a likely Nash equilibrium outcome and models it according to Choice Waves. The manner in which probabilistic choice of individuals in the marketplace can be modeled by Choice Waves provides insight into the underlying nature of their respective utility maximizing decision strategies.

## BACKGROUND

Certain human interactions have an overt, conscious component and a subconscious part. The individual player is aware of the components of the conscious part, but unaware of the subconscious part. Transactional analysis is a means within behavioral science of categorizing such interactions. Games within TA terminology are specifically the subconscious component. Subconscious interaction between individuals takes place on three principal levels: “parent,” “adult,” and “child.” All three of those “persons” serve a specific purpose and exist to one degree or another within everyone. The “adult” level is necessary for survival and is the state that gathers, processes, and analyzes data for dealing with the outside world. The “parent” level helps automate decisions, e.g., “This is the way we always do things.” The “child” level provides intuition, creativity, spontaneity, and enjoyment.

The interaction between two individuals interact on a subconscious level may be matched or mismatched, i.e., on the same level or on different levels. If matched interaction takes place (parent-parent, adult-adult, or child-child), then effective communication results. If the interaction between two individuals is not on the same level, then communication is hindered. Examples of mismatched communication include parent-adult, in which one player is in the parent level and the other is in the adult level. In such a case, the player in the adult state may feel as if he is being treated like a child or controlled by the player in the parent state. Communication states are not static and may in fact change during the course of a transaction from, for example, a child-child interaction to a parent-child or an adult-parent. Such level imbalances can cause disturbances in the outcome of even the simplest of transactions (Berne 1964). Thus, the subconscious state of economic actors has the potential to impact directly the outcome of strategic interactions within the economy.

Subconscious games, and hence decision strategy may be influenced by religious and moral beliefs, which are part of the categories of procedures and rituals in TA (Hawtrey and Johnson 2010; Oslington et al. 2011; Calomiris 2001; Johnson 2009; Johnson 2013). Broad social beliefs, which can be quite dynamic, may also influence decision strategies (Golan et al. 2001; Teisl, Roe, and Hicks 2002; Johnson et al., 2011).

## THE GAME

The colorfully-named "Now I've Got You" (NIGY) is a subconscious TA game that results when one player has the other player at his mercy, and the very state of having that other player at his mercy is more important to the player psychologically than any real or potential gains from the strong position. An example given by Berne is excerpted below from *Games People Play:*

> White needed some plumbing fixtures installed, and he reviewed the costs very carefully with the plumber before giving him a go-ahead. The price was set, and it was agreed that there would be no extras. When the plumber submitted his bill, he included a few dollars extra for an unexpected valve that had to be installed – about four dollars on a four-hundred-dollar job. White became infuriated, called the plumber on the phone, and demanded an explanation. The plumber would not back down. White wrote him a long letter criticizing his integrity and ethics and refused to pay the bill until the extra charge was withdrawn. The plumber finally gave in.

In the above example from Berne's work, both White and the plumber were playing games. The plumber was clearly the party in the wrong because a promise was made that the price was firm and would not change. White, however, did not choose to discuss the matter on the Adult level. Instead, he played NIGY, using the situation set up by the plumber, and reacted with enraged fury, attacking the plumber's entire business and personal philosophy and ethics. Had the interaction taken place on the Adult-Adult level, the problem of the disputed sum of money could have simply been pointed out and discussed calmly and in a dignified manner. However, since White played NIGY, a Parent-Adult interaction resulted as White exploited his superior position of being in the right. One possible rationale for White playing NIGY could be that it served as an outlet for years of pent-up frustration and various wrongs, real or perceived, done to him by others. It could be that his own mother or father may perhaps have done the same thing to him when he was young (Berne 1964).

When one players plays NIGY, the victim is forced to react in one way or another. For the plumber, utility maximization outside the presence of NIGY is almost definitely going to be different from the utility maximization strategy that must be adopted when White plays NIGY. Thus a Nash equilibrium situation is set up. As each player approaches a transaction, it is possible that one, both, or neither of the players have the potential to play the game, and it is similarly possible that one, both, or neither of the players have the potential to provide the trigger that will induce the other player to play NIGY. Thus the possible interactions between two players, A and B, are as follows:

1. A plays NIGY. B provides the trigger.
2. B plays NIGY. A provides the trigger.
3. A has the potential to play NIGY. B does not provide the trigger.
4. B has the potential to play NIGY. A does not provide the trigger.
5. A and B both have the potential to play NIGY. Neither provides the trigger.
6. Neither has the potential to play NIGY.

In each of the six possibilities above, there are different payoffs to each player. However, the utility-maximizing strategy of player A depends on the action of B, which depends on the strategy of B – and vice versa. In situations 1, 3, and 5 above, A is an NIGY player. The only difference is whether or not B provides the trigger. Thus A's strategy is two-part and contingent on the observed behavior of B. A maximizes utility by not playing NIGY if B does not provide the trigger, but maximizes utility by playing NIGY if B does provide the trigger. Even though the utility gained from playing NIGY is greater than when not playing NIGY, there is a penalty for playing NIGY when the trigger has not been provided. Thus there is no *a priori* dominant strategy for A in situations 1, 3, and 5. Player B maximizes utility through some decision strategy, which would typically mean not providing the trigger since there is a penalty to B if the trigger is provided and A plays NIGY. While that would suggest that B's dominant strategy is not to provide the trigger, sometimes B may be playing another game in which utility is gained by, for example, providing the trigger like a "chip on the shoulder" and hoping someone will take the bait (or perhaps enjoying the perceived thrill of not knowing whether or not the other player is an NIGY player). Player A, though, does not know *a priori* what type of player B is – and likewise neither does B know what type of player A is.

Assuming that B is not playing additional games, then B does have a dominant strategy of not providing the trigger, because in all cases B would be better off. Now, in the exemplar provided in the Berne text, the plumber was in fact playing a game, and hence the trigger was provided even not knowing what type of player White was. Nevertheless, in the absence of such additional games, B's dominant strategy remains not providing the trigger. Player A does not have a dominant strategy, but given that B will always refrain from providing the trigger, A will not play NIGY. That outcome is the Nash equilibrium of that game under that specific set of conditions. The specific outcome, however, depends on the probabilities associated with each player following a particular strategy. In the case of A, there may be a probabilistic nature to the strategy of NIGY, i.e., even in the presence of the trigger, the NIGY game will not always be played because of some factor. The factor could simply be a momentary whim not to play, or it could be an instant assessment of B such that it is believed by A that playing NIGY with B, even though B provided the trigger, will not result in a better outcome, and therefore utility is maximized by not playing. In the case of B, even in the absence of other games, there is a theoretical possibility that, for whatever reason at the decision point, B will provide the trigger. If other games are introduced to the environment, then there is a probability associated with B playing one or more other games that might lead to the trigger being provided. Additionally, B has a variety of response strategies or games that could be introduced as options that could be employed as a reaction to A playing NIGY, therefore introducing a probability that B no longer has a dominant strategy in the absence of other games to avoid the trigger.

## A CHOICE WAVE PROBABILISTIC MODEL

In the framework in which NIGY is a possibility, each player faces a utility function relevant to a particular good or decision choice that is a function itself of a Choice Wave that is a linear combination of that player's general Choice Wave related to the good or decision, the player's own Choice Wave pertaining to NIGY, and the other player's Choice Wave pertaining to whether or not they will provide the trigger and how they will respond if NIGY is played (Johnson 2011). Eqns. 1 and 2 are the Choice Waves for two players, denoted as A and B, regarding the game NIGY.

$$(1)\quad {}_{NIGY}\psi_A = \begin{cases} \text{Prob}^*\left(NIGY \middle| trigger_B\right) \text{ at the decision point;} \\ \text{Prob}\left(NIGY\right) \text{ s.t. } \text{Prob}_B\left(trigger\right) \text{ otherwise.} \end{cases}$$

$$(2)\quad {}_{NIGY}\psi_B = \begin{cases} \text{Prob}^*\left(NIGY \middle| trigger_A\right) \text{ at the decision point;} \\ \text{Prob}\left(NIGY\right) \text{ s.t. } \text{Prob}_A\left(trigger\right) \text{ otherwise.} \end{cases}$$

In Eqns. 1 and 2, when not at a decision point, i.e., not at a moment of transaction, then the probability that each player will play NIGY is subject to the probability that the other player will provide the trigger. So, the Choice Wave of A is subject to the Choice Wave of B. At the decision point, B has revealed whether trigger will be provided ($trigger_B$=1) or will not be provided ($trigger_B$=0) such that A's probability of playing NIGY is now based on that specific outcome rather than the probability of that outcome. Conversely, at the decision point, A has revealed whether trigger will be provided ($trigger_A$=1) or will not be provided ($trigger_A$=0) such that B's probability of playing NIGY is a function of that specific outcome. The full Choice Wave for each player, then, is given as Eqns. 3 and 4.

$$(3)\quad \psi_A = {}_A\lambda_1\, {}_x\psi_A + {}_A\lambda_2\, {}_{NIGY}\psi_A + {}_A\lambda_3\, {}_{Trigger}\psi_B$$

$$(4)\quad \psi_B = {}_B\lambda_1\, {}_x\psi_B + {}_B\lambda_2\, {}_{NIGY}\psi_B + {}_B\lambda_3\, {}_{Trigger}\psi_A$$

Now consider demand for a specific bundle of goods, *x*, in market in which there may exist NIGY players and others who provide NIGY triggers. That framework, other things being equal, yield four basic Consumer Sub-Types as follows:

Type N: NIGY Player
Type T: Trigger Provider
Type R: Does not play NIGY ( ${}_{NIGY}\psi = 0$ )
Type Q: Does not provide a trigger ( ${}_{Trigger}\psi = 0$ )

Each consumer is necessarily a composite of exactly two of the sub-types. Thus the Consumer Types in the market are as follows:

Type NT: NIGY Player & Trigger Provider
Type NQ: NIGY Player; does not provide a trigger
Type RT: Does not play NIGY; Trigger Provider
Type RQ: Does not play NIGY; Does not provide a trigger

In a market with the above Consumer Types, the transactional interactions that might result in a game of NIGY taking place are:

NT – RT
NT – NT
NQ – RT

NQ – NT

The market may be conceptualized as a number of individuals, represented as particles in a finite space in which the particles may move freely. When individuals interact for a transaction, it is equivalent to a “collision” between two particles. There is a probability, therefore, that any two individuals will interact. The nature of their transaction depends on the Consumer Type of each individual. The probability that an individual of one Consumer Type will interact with an individual of another particular Consumer Type is a function of the numbers of individuals in each Consumer Type, the total number of individuals in the market, and the relative effective distance between that individual and the nearest individual of the other Consumer Type (Johnson, 2015). This may be expressed as Eqn. 5.

$$(5)\ \ \text{Prob}\left(i\|j\right)=\frac{f\left(N_{NT},N_{RT},N_{NQ},N_{total}\right)}{g\left(r_{ij},r_{ik},r_{il},r_{im}\right)}$$

Eqn. 5 expresses the probability of an interaction between individual *i* and *j,* where *f* is a function of the numbers of the three Consumer Types that may lead to a an NIGY game and the total number of individuals in the market, and *g* is a function of the distances between *i* and *j,* and the distance between *i* and the nearest of each of the other Types, with one of the arbitrary terms *k, l,* and *m* being of the same type as *i.* The function *g* is expressed as a function of the effective distance between *i* and the nearest of the other possible types, not just to *j,* because there is a probability that individual *i* will encounter some other type before it interacts with *j.* If that happens, then at a given decision point, there will clearly be no interaction between *i* and *j.* The distance relationship is clearly inverse since the likelihood of an interaction is greater given a greater proximity. And, the proximity is, again, the “effective distance,” which need not be a physical distance, but can refer to the degree of influence. Through the internet, for example, one may interact more with someone on the other side of the world than with one’s next-door neighbor.

In Eqn. 5, *f* and *g* are arbitrary functional forms. However, for purposes of example, if *f* were to be a simple linear function and *g* were to be a quadratic function, then a possible form for Eqn. 5 for the specific case of an NT interaction with an RT could be Eqn. 5a.

$$(5a.)\quad \text{Prob}\left(NT\|RT\right)=\frac{\left(\dfrac{N_{NT}+N_{RT}+N_{NQ}}{N_{NT}+N_{RT}+N_{NQ}+N_{RQ}}\right)}{\left(r_{NT-RT}\right)^2+\dfrac{1}{\left(r_{NT-NT}\right)^2+\left(r_{NT-NQ}\right)^2+\left(r_{NT-RQ}\right)^2}}$$

In Eqn. 5a, the closer an NT player is to an RT player, the closer their effective distance, and so the larger its effect on the probability of interaction between those two probability types. That is offset by the effective distances of the other possible combinations such that the closer the NT player is to any of the other types, the smaller is the probability of an interaction. To choose some numbers for purposes of example, let $r_{NT-RT}=1$ and the other three distance terms equal 10, 2, and 3 respectively. Their distances are all greater than 1, and the denominator of Eqn. 5a becomes 1 + 1/16. If, on the other hand, $r_{NT-RT}=1$ and the other three distance terms equal 0.1, 0.2, and 0.2 respectively, then they are all closer to the NT player than is the RT player. In that case, the denominator of Eqn. 5a becomes 1 + 1/0.5 = 3, and so the probability of interaction is reduced because the relative distance to RT is less than the relative distances to the other player types. This is merely a set of examples of the many possibilities in both value and functional form. However, it illustrates that not only are the absolute numbers of each player type relative to the total number of individuals, but also both the absolute effective distance and relative effective distance are important in determining probability.

If there exist both individuals that have the capacity to play NIGY and those that have the capacity to provide the trigger, it is possible that the Nash equilibria resulting from such probabilistic interacts may influence the overall demand for a particular bundle of goods in the market. Empirical demand analysis in such a framework is providing an average of the demand in the absence of NIGY and the demand in the sub-market in which NIGY has taken place. Demand, then, should be expressed as an expectation value. Following Johnson (2016), the expectation value of demand for a given bundle of goods, *x,* can be expressed in the first instance as Eqn. 6.

(6) $$\langle x\rangle = p\left(h_1(\psi_{NT})\widehat{NT}+h_2(\psi_{NQ})\widehat{NQ}+h_3(\psi_{RT})\widehat{RT}\right)+h_4(\psi_{other})$$

In Eqn. 6, *p* is a probabilistic function determining the probability that a game of NIGY results, and the four functions *h* are some arbitrary demand functions. The term $\psi_{other}$ refers to the demand component resulting from Consumer Types that will not result in a possible game of NIGY. It is the first three functions within $\langle x\rangle$ that represent a deviation from what otherwise would be predicted not only by quasi-rationality (Russell and Thaler, 1985), but also by Choice Wave Probabilistic Demand, and they only come into play if there is in fact a game of NIGY. The Choice Waves of each individual Consumer Type do not explicitly determine whether there is a game of NIGY, but rather govern the individual decision strategy of the individuals within that Consumer Type. If the Choice Wave of an individual that can play NIGY or provide the trigger is a linear combination of the Choice Wave pertaining to the game and/or trigger *and* a Choice Wave of regular consumer choice, then the components of the individual's choice that do not relate to the game NIGY will be necessarily contained within $\psi_{other}$, for with respect to decision strategy not pertaining to the game, they are no different than the non-game Consumer Types in the market. If there are no interactions of individuals leading to a game of NIGY, then those individuals effectively behave as the individuals represented by $\psi_{other}$.

Following Johnson (2012), Eqn. 6 can be rewritten using a specific Choice Wave probabilistic demand model as Eqn. 6a.

(6a.)

$$\langle x\rangle = p\left(\frac{\frac{C_nM_n}{n\pi p_x}\int_0^I e\left|_{NT}\psi_n(e)\right|^2 de\widehat{NT}+\frac{C_{n'}M_{n'}}{n'\pi p_x}\int_0^I e\left|_{NQ}\psi_{n'}(e)\right|^2 de\widehat{NQ}+\frac{C_{n''}M_{n''}}{n''\pi p_x}\int_0^I e\left|_{RT}\psi_{n''}(e)\right|^2 de\widehat{RT}}{M_n+M_{n'}+M_{n''}}\right)$$
$$+\frac{C_{n'''}M_{n'''}}{n'''\pi p_x}\int_0^I e\left|_{Other}\psi_{n'''}(e)\right|^2 de\widehat{Other}$$

In Eqn. 6a, the values *n, n',* and *n''* represent the specific wave state of the Choice Wave itself associated with the specific Consumer Types NT, NQ, and RT, and the value *n'''* represents the Consumer Type "all other consumers." (Recall that, should a game of NIGY not take place, the NT, NQ, and RT consumers are assumed to behave as any other consumer. Hence the non-game component of their choice decision strategy is contained within the Choice Wave for all other consumers. The terms $C_n$, $C_{n'}$, $C_{n''}$, and $C_{n'''}$ are constants used as weighting terms. The terms $M_n$, $M_{n'}$, $M_{n''}$, and $M_{n'''}$ are size variables proper to each Consumer Type. In the integrals, $I$ is the maximum level of expenditure and hence is the budget constraint stemming from the underlying Market Potential Function that defines the space over which a consumer may make a consumption choice (Johnson 2012). As previously stated, $p$ is an arbitrary probability function giving the probability that a game of NIGY results, for that is a necessary condition for any of the terms shown in Eqn. 6a within that $p$ function to influence outcome at all.

The Choice Waves give the probabilistic decision strategy of each Consumer Type for choosing a particular good $x$. Thus, considering only the terms in Eqn. 6a that could result through interaction in an NIGY game, i.e., NT, NQ, and RT, the expectation value of their choice of $x$ is based first on their Choice Wave and then on the probability that an NIGY game takes place at all.

Note that each of the Choice Waves of the three Consumer Types that might yield an NIGY game exist in their own space orthogonal to the spaces of all other Consumer Types (Johnson 2012). However, the essence of transactional analysis game theory is that a transaction has in fact taken place. Therefore, transactions can only take place in this framework at the point at which the vector of an individual of one Consumer Type intersects the vector of the Consumer Type of the other individual with which the interaction took place. A game of NIGY *may* result if the space of one of the two Consumer Types with the N sub-type intersects that of one of the two Consumer Types with the T sub-type. Re-writing Eqn. 5 to yield Eqn. 7 gives an expression of the probability that a NIGY game will take place.

$$\begin{aligned}\text{Prob}\left(N\|T\right) = {} & \sum_{j=1}^{n_{NT}}\sum_{i=1}^{n_{RT}}\frac{1}{g_1\left(r_{NT_j,RT_i}\right)} + \sum_{j=1}^{n_{NT}}\sum_{i=1}^{n_{NT}-1}\frac{1}{g_2\left(r_{NT_j,NT_i}\right)} + \\ & \sum_{j=1}^{n_{NQ}}\sum_{i=1}^{n_{RT}}\frac{1}{g_3\left(r_{NQ_j,RT_i}\right)} + \sum_{j=1}^{n_{NQ}}\sum_{i=1}^{n_{NT}}\frac{1}{g_4\left(r_{NQ_j,NT_i}\right)}\end{aligned} \quad (7)$$

Eqn. 7 proposes that the probability of an N-sub-type interacting with a T-sub-type is the sum of the probabilities of each individual of each type interacting with each other individual of their own or other types. That probability is given as the inverse of a function of the effective distance between those two individuals. In Eqn. 6, the first three terms only in fact exist if there is a game of NIGY. Otherwise there is no impact to demand. Since Eqn. 7 gives the probability of an NIGY game, Eqn. 6 can be modified to be Eqn. 8.

$$\left\langle x\right\rangle =\sum_{j=1}^{n_{NT}}\sum_{i=1}^{n_{RT}}\frac{1}{g_1\left(r_{NT_j,RT_i}\right)}\left\{\left(h_1\left(\psi_{NT}\right)\widehat{NT}\right)\times\left(h_3\left(\psi_{RT}\right)\widehat{RT}\right)\right\}+$$
$$\sum_{j=1}^{n_{NT}}\sum_{i=1}^{n_{NT}-1}\frac{1}{g_2\left(r_{NT_j,NT_i}\right)}\left\{\left(h_1\left(\psi_{NT}\right)\widehat{NT}\right)\bullet\left(h_1\left(\psi_{NT}\right)\widehat{NT}\right)\right\}\widehat{NT}+$$
$$\sum_{j=1}^{n_{NQ}}\sum_{i=1}^{n_{RT}}\frac{1}{g_3\left(r_{NQ_j,RT_i}\right)}\left\{\left(h_2\left(\psi_{NQ}\right)\widehat{NQ}\right)\times\left(h_3\left(\psi_{RT}\right)\widehat{RT}\right)\right\}+ \quad (8)$$
$$\sum_{j=1}^{n_{NQ}}\sum_{i=1}^{n_{NT}}\frac{1}{g_4\left(r_{NQ_j,NT_i}\right)}\left\{\left(h_2\left(\psi_{NQ}\right)\widehat{NQ}\right)\times\left(h_1\left(\psi_{NT}\right)\widehat{NT}\right)\right\}+$$
$$h_4\left(\psi_{other}\right)$$

In Eqn. 8, because an interaction between two individuals of different Consumer Types represents an intersection between two orthogonal vectors in hyperspace, the interactions are represented as cross products. This yields a new vector that is orthogonal to the vectors of each consumer type, i.e., a new Choice Wave results that is the vector product of their interaction. Conceptually, this means that the impact to the expectation value of demand of *x* from the NIGY game is a resultant effect of an interaction between players. The exception to the application of a cross product is when an NT interacts with another NT. That is represented as a dot product in Eqn. 8 for the purpose of generality, and the resulting vector will remain in the direction of the original NT vector. However, that may be replaced by a cross product if one particular assumption is adopted, and that is if an NT interacts with an NT, in some way the fact that they both provide the trigger and both are NIGY players cancels each other out, i.e., NIGY cannot be played by both players in a two-player transaction. In such a case a cross product is appropriate since the resultant vector magnitude would be zero. For that to be a valid assumption, however, both players would have to have a dominant strategy of not playing NIGY. For that to be the case, they would have to have prior knowledge of each other's Consumer Type and accompanying decision strategy. Since that violates key assumptions, the only way for that to be practically plausible would be for each to be a "high perceptive" player, i.e., one who is able to discern the decision strategy of the other player a high percentage of the time. The concept of a highly perceptive player is not limited to discerning others from one's own Consumer Type. These possibilities, however, represent special cases. Thus Eqn. 8 is expressed in general conceptual terms to be as broadly applicable as possible. However, Eqn. 6a can be substituted into Eqn. 8 to yield a specific example, Eqn. 8a.

(8a.)

$$\langle x\rangle=\sum_{j=1}^{n_{NT}}\sum_{i=1}^{n_{RT}}\frac{1}{g_1\left(r_{NT_j,RT_i}\right)}\left\{\left(\frac{C_nM_n}{n\pi p_x}\int_0^I e\left|_{NT}\psi_n\left(e\right)\right|^2 de\widehat{NT}\right)\times\left(\frac{C_{n''}M_{n''}}{n''\pi p_x}\int_0^I e\left|_{RT}\psi_{n''}\left(e\right)\right|^2 de\widehat{RT}\right)\right\}+$$

$$\sum_{j=1}^{n_{NT}}\sum_{i=1}^{n_{NT}-1}\frac{1}{g_2\left(r_{NT_j,NT_i}\right)}\left\{\left(\frac{C_nM_n}{n\pi p_x}\int_0^I e\left|_{NT}\psi_n\left(e\right)\right|^2 de\widehat{NT}\right)\bullet\left(\frac{C_nM_n}{n\pi p_x}\int_0^I e\left|_{NT}\psi_n\left(e\right)\right|^2 de\widehat{NT}\right)\right\}\widehat{NT}+$$

$$\sum_{j=1}^{n_{NQ}}\sum_{i=1}^{n_{RT}}\frac{1}{g_3\left(r_{NQ_j,RT_i}\right)}\left\{\left(\frac{C_{n'}M_{n'}}{n'\pi p_x}\int_0^I e\left|_{NQ}\psi_{n'}\left(e\right)\right|^2 de\widehat{NQ}\right)\times\left(\frac{C_{n''}M_{n''}}{n''\pi p_x}\int_0^I e\left|_{RT}\psi_{n''}\left(e\right)\right|^2 de\widehat{RT}\right)\right\}+$$

$$\sum_{j=1}^{n_{NQ}}\sum_{i=1}^{n_{NT}}\frac{1}{g_4\left(r_{NQ_j,NT_i}\right)}\left\{\left(\frac{C_{n'}M_{n'}}{n'\pi p_x}\int_0^I e\left|_{NQ}\psi_{n'}\left(e\right)\right|^2 de\widehat{NQ}\right)\times\left(\frac{C_nM_n}{n\pi p_x}\int_0^I e\left|_{NT}\psi_n\left(e\right)\right|^2 de\widehat{NT}\right)\right\}+$$

$$\frac{C_{n'''}M_{n'''}}{n'''\pi p_x}\int_0^I e\left|_{Other}\psi_{n'''}\left(e\right)\right|^2 de\widehat{Other}$$

## CONCLUSIONS

Through the application of Choice Wave Probabilistic Demand, it is possible to see a mathematical framework in which demand for a bundle of goods is influenced by the presence of individuals in the marketplace that play subconscious transactional games. Game players in sufficient numbers introduce new Consumer Types into the market with Choice Waves that are linear combinations of the “typical” consumer decision Choice Wave and additional Choice Waves that relate to probabilities of playing a particular game at the point of a transaction. The more such individuals exist in the market and the higher the probability is that players of a particular game will interact with others who provide a trigger for a game (if a trigger is indeed necessary), the more likely it is that the expectation value of demand will be influence by their subconscious games.

The subconscious games also lead to transactions with no knowledge or limited knowledge *a priori* about the other individual’s decision strategy. It is only when a transaction takes place that the information about the other player is revealed. That information in turn may influence one’s own decisions. Thus is becomes a probabilistic concept of strategic maximization of utility based on the actions of others each individual may encounter. As that itself is probabilistic, one’s own strategy is necessarily probabilistic, therefore leading to situations in which there are reactive Nash equilibria and not always a dominant strategy.

In empirical estimation, the results from general data sets may therefore be skewed. Since the games are subconscious, however, and they do not reveal themselves until the moment of transaction, but may not be obvious to others or be obvious within the data set, testing for the presence of subconscious games using typical economic data is difficult. Behavioral experimentation on the type and number of individuals in a market that may play various subconscious games can be useful to estimating the number of such individuals that may be present when conducting empirical economic analysis.

Transactional Analysis games provide good examples of human strategic interaction that often ends in a Nash equilibrium. Transactional Analysis (TA) was established as a branch of behavioral science in the 1960s by Eric Berne, an American medical doctor. This study considers strategic interaction between two players of a specific TA game known as "Now I've Got You" that has a likely Nash equilibrium outcome and models it according to Choice Waves. The manner in which probabilistic choice of individuals in the marketplace can be modeled by Choice Waves provides insight into the underlying nature of their respective utility maximizing decision strategies.

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