Filling *nā puka*[1] with PUFM: Empowering teachers with profound understanding of fundamental mathematics

Eula Ewing Monroe
Brigham Young University Provo

John Bailey
Brigham Young University Hawaii

Barry Mitchell
Brigham Young University Hawaii

Yvonne AhSue
Laie Elementary School, Laie, HI

ABSTRACT

This paper reports results from a Mathematics and Science Partnership grant designed primarily to help teachers from six elementary schools on the North Shore of O‘ahu develop a profound understanding of fundamental mathematics (PUFM) (Ma, 1999). Five of these schools had not met minimum requirements under the No Child Left Behind Act; among them there were also 19 teachers not fully licensed in Hawaii, therefore not meeting the definition of “highly qualified teacher.”

Thirty-three teachers self-selected to participate for 1, 2, or all 3 project years ($\bar{X}$ per year, $18.6\overline{6}$). Grade levels taught ranged from Kindergarten through grade 6, with 31 regular education teachers and 2 special educators.

To help teachers develop PUFM, the project focused on increased mathematics content knowledge while simultaneously addressing

- changed teacher beliefs in the direction of *Standards*-based mathematics education (NCTM, 1991, 2000);
- pedagogical practices focusing on student development of mathematical reasoning and problem solving via discourse-based instruction; and
- evidence of measurable/observable student gains in reasoning and problem solving.

The extent to which the project met its overall goals is described in reports available from baileyj@byuh.edu.

This paper reports progress toward the following specific outcomes: (a) changed teacher beliefs; (b) mathematics content knowledge addressed during the third year of the project; and (c) transformations in teacher practice. The first two outcomes were selected primarily to share instrumentation that may be helpful to other professional development researchers. The third outcome provides insights into understanding and contextualizing the realities of teacher change.

Keywords: elementary mathematics, professional development, assessment, conceptual understanding

INTRODUCTION

A foremost concern for teachers of elementary students is that they "must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (NCTM, 2000, p. 17). Mewborn (2003) reported:

> An abundance of research tells us that many teachers do not possess this deep and rich knowledge of mathematics. This body of research overwhelmingly paints a dismal picture of teachers' conceptual knowledge of the mathematics they are expected to teach. (p. 47)

In a seminal study of elementary classrooms in the United States and in China, Ma (1999) found that this kind of deep knowledge was virtually absent in elementary classrooms in the United States. She found that the critical difference between teachers in the two countries was that Chinese teachers were much more likely to have what she termed "a profound understanding of school mathematics" (p. 124). This profound understanding of fundamental mathematics, or PUFM, is defined as follows:

> Profound understanding of fundamental mathematics (PUFM) is more than a sound conceptual understanding of elementary mathematics—it is the awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual structure and instill those basic attitudes in students. (p. 124)

Teachers who have PUFM "know how, and also know why" (p. 108); they are enabled to promote multiple approaches to problem solving and connections among mathematics concepts and procedures. Such teachers also have an understanding of the elementary mathematics curriculum as a unified body of knowledge, knowing the central ideas that must be revisited and reinforced and those that that lay the groundwork for concepts to be studied later.

The body of research evidence from a number of professional development projects suggests that enhancing teachers' mathematical knowledge is a crucial part of learning to teach students effectively (Mewborn, 2003). According to Schifter (as cited in Mewborn, 2003):

> Teachers need to have opportunities to learn mathematics in the ways in which they are expected to teach it to students. They need to struggle with important mathematical ideas, justify their thinking to peers, investigate alternative solutions proposed by others, and reconsider their conceptions of what it means to do mathematics. In short, teachers' thinking needs to be at the center of professional development sessions just as children's thinking needs to be at the center of mathematics instruction. Teachers need to revisit the mathematics they are teaching to gain insights into the conceptual underpinnings of the topic and the interconnections among topics. (p. 39)

Nevertheless, just as mathematics content knowledge apart from knowledge of other aspects of teaching and learning typically does not transform teachers' daily classroom practice, such content knowledge is not nearly as effective if developed in isolation from other aspects of learning (NCTM, 2007), aspects considered necessary for PUFM. Wood, Nelson, and Warfield (2001) indicated that teacher change occurs in three domains—in teachers' beliefs, knowledge, and practice. To effect transformative change, teachers must have opportunities to learn not only mathematics content but also the content knowledge for teaching, or pedagogical content knowledge, as well as understanding of student thinking and development (e.g., Bahr, Monroe, & Wentworth, 2010; Wood, Nelson, & Warfield, 2001). Further, they must integrate their understanding of these knowledge components as they plan, implement, and assess instruction.

Such teacher learning can be facilitated through carefully developed, thoughtfully implemented, sustained professional development (NCTM, 2007).

This paper describes selected elements of a 3-year professional development project that had as its primary purpose helping regular elementary teachers and special educators develop a profound understanding of fundamental mathematics (Ma, 1999). Professional development tasks focused on helping teachers fill *nā puka*[1], or holes, in their content knowledge of mathematics while concurrently addressing pedagogical content knowledge and understanding of student thinking and development. Transfer into practice was facilitated by ongoing lesson study. (Full reports of this project are on file with the Hawaii Department of Education and are available from baileyj@byuh.edu. See Appendix A for a picture of the model that depicts the essential elements that guided ongoing work with the participants.)

PROJECT DESCRIPTION

During 2004-2007 the authors were involved in a Mathematics and Science Partnership grant to support a group of self-selected regular elementary teachers and special educators from six target schools on the North Shore of Oʻahu in developing a profound understanding of fundamental mathematics (PUFM) (Ma, 1999). These schools are part of the Windward School District, which has more than 10,000 students from diverse backgrounds, with at least 50% entitled to Title I benefits; schools are impacted from a low of 20% to a high of 75.6%. Five of the six target schools identified for this project had not met minimum requirements under the No Child Left Behind Act and were substantially below state averages in the latest available Hawaii Content and Performance Standards statewide test results prior to the project. Furthermore, the participating schools had among them 19 teachers who had not met the full licensing requirements for the State of Hawaii and therefore did not meet the definition of "highly qualified teacher."

The overarching goal for this project was to help these teachers develop a deep and connected knowledge of the mathematics taught in the elementary school curriculum. This kind of knowledge, or PUFM (Ma, 1999), is notably limited in many American teachers (Ma, 1999; Ball, Thames, & Phelps, 2008). The teachers involved in this project were no exception to this pattern, yet they were eager to learn. During the 3 years of the project, a total of 33 regular education and special education teachers participated according to the following schedule:

Summer	10 days (60 clock hrs)
Fall	3 days (18 clock hrs)
Winter	Classroom/school visits and approximately 3 hours of group work
On a needs/request basis	Consulting with individuals or small groups of teachers

Of the teachers involved in the project, 6 participated for the entire project, with 11 participating for two of the project years and 16 participating for one of the years ($\bar{X}$ per year, $18.6\bar{6}$). These teachers ranged in grade levels taught from Kindergarten through grade 6 and were primarily from three elementary school faculties (School A: 12 participants; School B: 9 participants, and School C: 5 participants), with the remaining 7 being sole participants from their schools. Of the participants, 31 were regular elementary teachers and 2 were special educators.

Within this context the authors of this paper learned about helping teachers recognize *nā puka* in their knowledge of the fundamental mathematics necessary for teaching elementary school students. At the same time they provided an environment in which these teachers were empowered to take steps to fill these gaps. However, as important as mathematics content knowledge is to effective instruction, it is not enough. Good teachers of mathematics must also possess other knowledge components, most notably pedagogical content knowledge and understanding of student thinking and development (e.g., Wood, Nelson, & Warfield, 2001). Further, they must integrate their knowledge of these components as they plan, implement, and assess instruction. Therefore, during the project the study of pedagogical content knowledge and understanding of student thinking and development were embedded in professional development tasks focused on helping teachers learn fundamental mathematics.

The content standards identified by the National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics* (PSSM) (2000)—Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability—guided the organization of the professional development activities. A combination of resources guided the workshop leaders' own pedagogy as well as the pedagogy they sought to teach, with the process standards identified in PSSM (NCTM, 2000) and the teaching standards included in *Professional Standards for Teaching Mathematics* (NCTM, 1991) serving as foundational ideas. To help teachers develop understanding of children's mathematical thinking and cognitive development, *Children's Mathematics: A Cognitively Guided Approach* (Carpenter, Fennema, Franke, Levi, & Empson, 1999) was a major resource. To summarize both the content and approach, the term *Standards*-based mathematics education (e.g., NCTM, 1991, 2000) was used.

The overall desired goals of the project were as follows:

- Increased teacher competence in the content of the Hawaii State K-6 mathematics curriculum
- Changed teacher beliefs in the direction of *Standards*-based mathematics education (NCTM, 1991, 2000)
- Pedagogical practices that focus on student development of mathematical reasoning and problem solving via discourse-based instruction
- Evidence of measurable/observable student gains in mathematical reasoning and problem solving on multiple measures of achievement

The success of the project in attaining the first three goals has been documented through both qualitative and quantitative measures; however, the level of success with the fourth goal remains inconclusive because of changes in the structure of the state assessment implemented to meet the requirements of No Child Left Behind legislation as well as the limitations of available criterion-referenced grade-level tests in making comparisons to determine annual yearly progress (e. g., Martineau, 2006). (Full reports of this project are on file with the Hawaii Department of Education and are available from baileyj@byuh.edu)

In this paper progress toward the following specific outcomes is reported:

- Measured changes in teacher beliefs
- Assessed growth toward our content emphases for Year 3: Data Analysis and Probability; Rational Number
- Transformations in practice as reported by one of the project participants (who also serves as a coauthor of this paper)

The first two outcomes are reported to share instrumentation that may be of value to researchers in studying growth toward similar professional development goals. The third outcome gives a "backward look" into the immediate and sustained effects of the professional development in which the reporting participant was involved. Using reflections and assignments created during the project as well as records of more recent academic and professional experiences as data sources, she provides insights that may be helpful in understanding and contextualizing the realities of teacher change.

ASSESSMENT OF TEACHER BELIEFS

Method

To examine changes in teacher beliefs about mathematics teaching and learning, the *Integrating Mathematics and Pedagogy (IMAP) Beliefs Survey*, an online instrument developed by Ambrose, Clement, Philipp, & Chauvot (2004), was used. (A browse version is available at http://www.sci.sdsu.edu/CRMSE/IMAP/pubs.html) The following seven beliefs have been found to support *Standards*-based mathematics education (NCTM, 1991, 2000) as investigated by Philipp et al., 2007.

Beliefs About Mathematics

1. Mathematics is a web of interrelated concepts and procedures (school mathematics should be too).

Beliefs About Learning or Knowing Mathematics, or Both

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts. That is, students or adults may know a procedure they do not understand.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's (Students') Learning and Doing Mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible. (Philipp et al., 2007, p. 65)

The *IMAP Beliefs Survey* assesses the intensity to which respondents possess these seven beliefs, using written or video cases to which teachers are asked to respond. The responses are then analyzed via rubrics that allow for inferences to be made about the intensity of the beliefs held by those taking the survey.

Results and Discussion

A Paired Student's t-test was used for pretest-posttest comparisons of participants' Beliefs scores. Pretest-posttest comparisons (see Table 1) provide evidence of significant positive change in beliefs. This change occurred in Beliefs 1 (t [23] = 3.01 [p < .01]), 3 (t [23]=1.87 [p <. 05]), 6 (t [23] = 3.43 [p < .01]), and 7 (t [23]=1.88 [p <. 05]), but no significant change at or below the .05 level in Beliefs 2, 4, or 5 (see Table 2).

Although the small sample size and the absence of a control group limit generalizability of the findings, the results indicate that for this group the overall beliefs (see Table 1) changed significantly in the direction of *Standards*-based mathematics education (NCTM, 1991, 2000). These results are also strongly supported by qualitative data from participant reflections and other written assignments. (See section of this paper labeled **"Closing comments: "We can do hard things!"** At the same time, some beliefs among this group of teachers appeared more resistant to change. Any explanation of this finding would be speculative; nevertheless, because of the small sample size, the effects found (p < .05) were deemed to be powerful.

ASSESSMENT OF DATA ANALYSIS AND PROBABILITY; RATIONAL NUMBER

Method

The content foci during the third and final year of the project were: (a) Data Analysis and Probability, the remaining NCTM content standard (2000) to be addressed in the project, and (b) Rational Number, chosen at the end of the second year because the workshop leaders and participants identified this content as an area of need. (Although the Number and Operations content standard had been addressed during the first year of the project, the emphasis had been on whole numbers and operations, to the neglect of rational number.) These content strands were assessed using selected *Diagnostic Teacher Assessments in Mathematics and Science* (DTAMS) (Brown, McGatha, & Karp, 2006), which examine a range of knowledge and processes of four types:

Type I–memorized/factual knowledge
Type II–conceptual understanding
Type III–reasoning/problem solving
Type IV–pedagogical content knowledge

Because this project focused primarily on mathematics content knowledge (Ma, 1999), which is viewed as inseparable in actual classroom practice from the pedagogical content knowledge of teachers (Ball, Thames, & Phelps, 2008), the DTAMS instruments selected for assessment were deemed appropriate. Two parallel forms of DTAMS instruments available for the domains specified were selected as assessment measures. These instruments, which contextualize the mathematics in word problems, are described below.

DTAMS Elementary Mathematics: Probability/Statistics/Algebra Assessment. This instrument was used to assess teacher learning in the Data Analysis and Probability content strand. Algebra was not a focus of the third project year; nonetheless, it was a component of the assessment measure. Thus it served as a benchmark against which effects of instruction could be compared. That is, if there were significant gains in Probability and Statistics, topics that received explicit attention, but no significant gains in Algebra, which received only incidental attention during this project year, gains could be interpreted as likely project effects.

DTAMS Elementary Mathematics: Rational Numbers Assessment. This instrument was used to assess teacher learning in the Rational Number content addressed during the third year of the project.

A Paired Student's *t*-test was used for pretest-posttest comparisons. (See Table 3.) Any effects found ($p < .05$) were deemed to be particularly powerful because of the small sample size.

Results and Discussion

For the Probability/Statistics/Algebra Assessment (see Appendix B for sample items), significant differences between overall pretest and posttest scores were found. Scores on the subtests for the two content areas explicitly targeted—Data Representation and Analysis; Probability—evidenced significant differences as well ($p < .01$). (See Table 3.) For the Algebra subtest, no significant difference was found between pretest and posttest scores. This result is not surprising; the content of Algebra was not specifically addressed during this year of the project.

For the *Rational Numbers Assessment*, no significant differences ($p < .05$) between pretest and posttest scores were found (see Table 4). This lack of significance may be explained by two factors. This topic, the last one addressed in the workshop, was the focus of instruction for only 18 class hours. Also, because of the extent of *nā puka* in participants' background knowledge, most of this instruction was provided at the concrete and pictorial level; overall, the test itself was more abstract than the instruction. Extensive additional work with rational numbers is recommended for the participants to be able to understand rational numbers more deeply at all levels—concrete, pictorial, and abstract. (See Appendix C for sample items.)

TRANSFORMATIONS IN PRACTICE

When, during the very first session of the project, participants introduced themselves with varying levels of apprehension about their involvement in a "math" project, the workshop leaders recognized that teacher affect would be a major factor to be addressed in ongoing planning and implementation for teacher change (e.g., Remillard & Bryans, 2004; Warfield, Wood, & Lehman, 2005). These teachers had volunteered to be involved, and, although they received professional development credit, their books and materials for the sessions, and limited funds for mathematics manipulatives for their classrooms, there were essentially no other tangible incentives to sustain them long enough to support them in changing their practice. The workshop leaders' concerns were substantially heightened when, during the first pre-assessment, more than one teacher ended up in tears. Nevertheless, they pressed forward, relying on established guidelines for professional development in mathematics education with which they were familiar. (See Appendix D for principles that participants identified during the first year of work in the project; these principles came to be known affectionately as "the Monroe doctrines" of professional development for the project.)

One of the participants, Yvonne—then a third grade teacher and one of the 6 who participated with virtually no absences or interruptions for the duration of the project, introduced herself as "Special Ed" in mathematics at the first session. During the first year of the project, she gained confidence and became empowered in her teaching practice. During a follow-up visit, she commented, "I taught my class the geometrical shapes and 70 percent of them passed the test, with 4 students having special needs scoring just under the 80 percent pass mark," and, on another occasion, "Now I know that kids are smart!"

The workshop leaders asked Yvonne to serve as a coauthor on this project because of the value her insights and perspective as a participant. She has checked the validity of the findings and inferences that are reported in this paper; she has also provided her story of change, as follows.

> Measuring a puddle was my first insight into recognizing the need for kids to explore. "You don't usually measure a puddle, but if you needed to, how would you go about it?" I wondered as I viewed a video demonstrating an inquiry-based lesson. These young students measured around it, the depth, and some even tried to soak it up with a sponge. I was in awe at how creative their thinking was and at the many different ways they thought to tackle this problem.
>
> I've been to countless workshops, classes, and conferences, sent home with my mind filled with optimism to make changes, but with no sense of where to start or how to bring about curricular goals. PUFM was different, for it was the kids who drove the instruction.
>
> I set out with one task and a large piece of butcher paper and a marker for every pair of students. I asked, "Johnny had three packages of gum. Each package had 5 sticks of gum. How many sticks of gum did Johnny have altogether?" Immediately I had 22 blank stares; then one brave girl asked, "What do we do?" "Solve it!" I said boldly, yet inside I wondered what to do next. Several students, ages eight and nine with no multiplication lessons to date, asked about the paper and the marker. I told them to use it to solve the problem. Used to workbooks, they seemed confused, yet willing. And soon the ball was rolling and they began drawing and talking. I saw packages of gum, I saw the five sticks, I heard them counting, and my heart was full. "It works!" I thought, "But what next?" I quickly went from group to group and asked if they could write a number sentence to go with their work. One group wrote, "3 x 5 = 15." I asked how they got it and they explained that it was five, three times. Several groups wrote 5 + 5 + 5 = 15. I asked them to count aloud. "Five, ten, fifteen," said one and another counted, "one, two, three, four, five; six, seven, eight, etc." With clipboard in hand, I quickly made note of the three levels of understanding. I also understood why some students may not be quite ready to move on, and in this case, to multiply. I felt certain that exploring tasks versus teaching algorithms would foster success for learners at every level.
>
> My next step was to implement a lesson plan template to make daily planning doable. I downloaded the PUFM template and found it to be unusual, yet helpful and thorough. The Launch was first, which asked to give a brief preparation (5-10 minutes) for the daily task. I did not teach a concept here; rather I provided experiences that assured me that the students were ready to solve the day's problem. For instance, one day I wanted the students to discover equivalent fractions, so I used my overhead projector to show a picture of a circle with two fourths shaded in. With a dry-erase board in hand, each student was to write a fraction to match the picture. I made note of those who were successful, those who saw one half, and those who did not quite see what I expected. Then I wrote a fraction and asked the students to make a drawing to match the fraction. I continued showing pictures and writing fractions and asking "true or false" questions, making notes each step of the way. By the end of 10 minutes I knew which students needed scaffolding, so as we moved into phase two of the lesson plan (Exploration) I made sure I provided questions that guided those students toward success. The third and last phase is the Summarize component of the lesson. I use the majority of this time to

allow teams to share what they have drawn and/or written on their butcher paper. Consequently it has become a time of assessment, another opportunity to teach peers, and a way to teach themselves—for those who learn through talking and through hearing other students' explanations.

Inquiry-based teaching has been a challenge for me, since there has been no resource or support within our school. At times I feel at odds, feeling as though I'm alone in my passion. However, one personal experience drives my endeavor to continue. It happened in the PUFM class while Dr. Monroe continued to scaffold us teachers to solve a word problem. We all stretched our brains, worked hard to solve this problem, and yet it remained unresolved. It was amazing how Dr. Monroe continued to ask us questions to keep our pursuit going. Forty-five minutes later we still had not resolved it, yet we felt closer to a resolution. Class ended and this problem stuck with me all through the night. By morning I figured it out, ran into class ready to show my answer; ready to take on more. Never before had I been so driven to complete a task. And the greatest joy. . . I had learned!

CLOSING COMMENTS: "WE CAN DO HARD THINGS!"

The overall data for the project (available from baileyj@byuh.edu) as well as the specific examples included in this report indicate that this project was successful in supporting teachers in the ongoing process of filling in *nā puka* in their preparation for teaching elementary school mathematics. At the same time, there were qualitative dimensions to the project that were not measured—and perhaps not measurable—that made the project professionally and personally satisfying to the workshop leaders and to the participants. The following discussion serves as an example.

Along the way, mathematics got difficult for each of the participants, as it does for everyone at some point. (Einstein is quoted as having said: "Do not worry about your difficulties in Mathematics. I can assure you mine are still greater" (iWise—Wisdom on Demand, n.d., p. 1). When the going gets difficult, the temptation to give up rather than to persevere often surfaces. To encourage themselves to persevere when faced with difficult mathematical concepts and ideas, the group adopted a motto: "We can do hard things!") Below are a few quotations from participants (shared anonymously) that will attest to their strong desire to persevere and to learn.

> I truly believe I have benefited from taking these classes the past 3 summers. I have gained so much knowledge . . . . I will take what I have learned and reflect on how I can be a better teacher. I appreciate all your patience, knowledge, love, and encouragement. I may not have a bright light bulb as others but I am capable of learning and understanding more. I can do hard work. Thank you . . . for all the support.
>
> The more classes I've taken the more my brain has hurt—but it has been a good hurt! I feel more confident—it's possible I might be ok teaching 6th [grade]. [I will] continue to listen more to children and tell myself, "I can do hard things."
>
> I am grateful for these years of math. I really appreciate your patience in helping me to learn all of those concepts. It has [been] a wonderful learning experience for me. I hope one day I will be just as enthusiastic about teaching math as you.

The authors of this paper, too, are grateful. They have been blessed with the opportunity to work with a wonderful group of dedicated teachers who are eager to learn the mathematics and related pedagogy, child development, and student mathematical thinking they need to help students in their classrooms succeed mathematically. Almost without exception, the teachers have gone the "extra mile" in preparation for sessions and in commitment to their own learning, the learning of their colleagues, and the learning of their students. The workshop leaders, too, have grown as professionals as they have challenged themselves to think in new ways to meet participant needs. Although the project has come to a close, the teachers involved represent a critical mass for providing classroom, grade level, and school-wide leadership for continuing the work of improving the mathematics education of students in Windward School District.

REFERENCES

Ambrose, R., Clement, L., Philipp, R., & Chauvot, J. (2004). A methodological approach for developing research rubrics to assess elementary school teachers' beliefs about mathematics and mathematics learning. *School Science and Mathematics, 104*(2), 56-69.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Bahr, D**.,** Monroe, & Wentworth, N. (2010). *A theoretical framework for providing and investigating reform-based professional development in mathematics*. Manuscript in progress.

Brown, E. T., McGatha, M., & Karp, K. (2006). Assessing teacher knowledge: Diagnostic assessments for elementary and middle school teachers. *New England Mathematics Journal, 28*(2), 37-50.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann and National Council of Teachers of Mathematics.

iWise—Wisdom on Demand. (n.d.). Retrieved March 16, 2010, from http://www.iwise.com/oL8wa

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics.* Mahwah, NJ: Lawrence Erlbaum.

Martineau, J.A. (2006). Distorting value added: The use of longitudinal, vertically scaled student achievement data for growth-based, value-added accountability. *Journal of Educational and Behavioral Statistics, 31,* 35-62.

Mewborn, D. S. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, W.G. Martin, & D Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 45-52). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2007). Mathematics teaching today: Improving practice, improving student learning (2nd ed.). Reston, VA: Author.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., et al. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38,* 438-476.

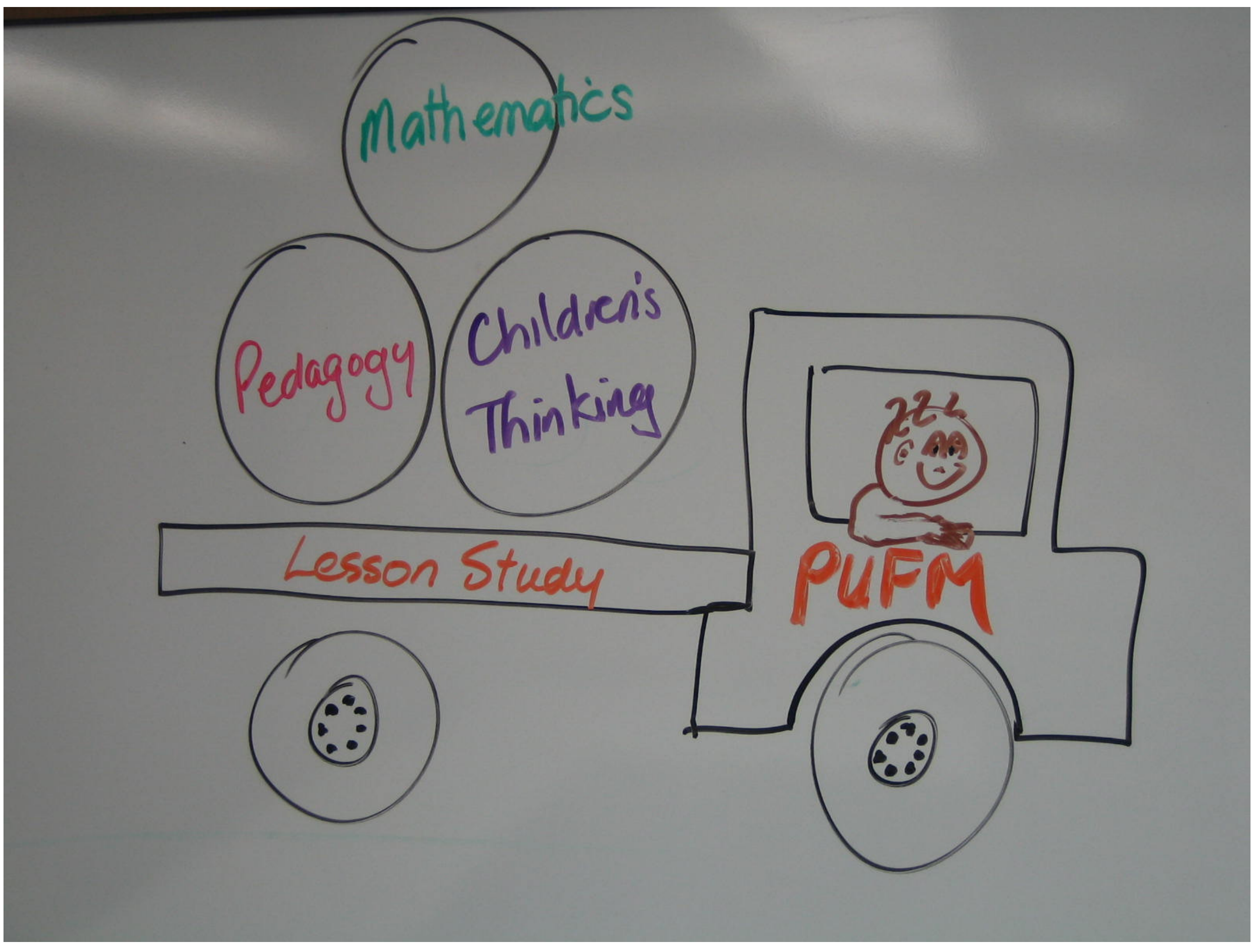
Remillard, J. T., & Bryans, M.B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*(5), 352-388.

Warfield, J., Wood, T., & Lehman, J. D. (2005). Autonomy, beliefs, and the learning of elementary mathematics teachers. *Teaching and Teacher Education, 21*(4), 439-456.

Wood, T., Nelson, B. S., & Warfield, J. (Eds.)**.** (2001). *Beyond classical pedagogy: Teaching elementary school mathematics.* Mahwah, NJ: Lawrence Erlbaum.

Appendix A

Model for our Day-to-Day Work with Participants



Appendix B

Sample Items for Diagnostic Teacher Assessments for Mathematics and Science (DTAMS), Probability/Statistics/Algebra Assessment.

Subcategory	Type 1 Memorized/ factual knowledge	Type 2 Conceptual understanding	Type 3 Reasoning/problem solving	Type 4 Pedagogical content
Probability	Which phrase below represents the likelihood of an event that will probably **not** happen? a. certain b. highly likely c. impossible d. highly unlikely	Yong played a carnival game in which she picked one ticket each from two different boxes. Each box contained one ticket marked "cat" and one marked "dog". [sic] To win, she must draw matching tickets. Which of the following gives a list of all possible equally likely outcomes (sample space) of this experiment? a. (cat, dog), (cat, cat), (dog, dog), (dog, cat) b. cat, dog c. cat, dog, cat, dog d. (cat, dog), (cat, cat), (dog, dog)	The best free-throw shooter on the Ewing Middle School girls' basketball team has a 50% chance of making the first shot percentages vary depending on the outcome of the first shot. If she makes the first shot, she has a 70% chance of making the second. However, if she misses the first shot, she only has a 40% chance of making the second shot. If she throws two free throws, what are the probabilities of making 0, 1, or 2 shots?	You are demonstrating a probabilistic event to third grade students. You use a coin, tossing it 10 times and have the students record the outcomes. The coin lands **heads** up on every toss. How would you explain this outcome to the students?
Data Representation & Analysis	Which of the following is the median for the data set: 4, 9, 3, 5, 10, 3? a. 3 b. 4 c. 4.5 d. 5.7	The graph* below shows the average salary of workers 18 years and older based on education. Which of the following is TRUE?	A national pizza company surveyed customers about their favorite pizza toppings. The graph* below shows the results. Students at Janson Middle School were surveyed about their	Your fifth grade class is studying sampling in a unit on data analysis. A student claims that she can generate a random sample of the numbers

		The difference in average salaries between these two groups… a. remains the same. b. Becomes greater as time goes on. c. Becomes less as time goes on. d. Doubles as time goes on.	favorite pizza toppings. a. If 63 students at Janson Middle School selected mushrooms as their favorite topping, how many would you predict would select pepperoni? Explain your answer. b. Why might the results from Janson Middle School be different from the national pizza company survey?	1, 2, 3, . . . 36 by repeatedly tossing a pair of dice and finding the product on the upturned faces of the dice. a. Describe the error in this student's strategy for generating a random sample. b. Describe a more valid strategy for creating a random sample of the numbers 1-36.
Algebraic Ideas	What are the coordinates of point A? a. (1, 3) b. (-3, -1) c. (-1, 3) d. (3, 1)	Sharon delivered 2 more cases of bottled water than DeMarcus and Torian delivered 3 times as many as Sharon. If DeMarcus delivered *n* cases of bottled water, which of these represents the number of cases that Torian delivered? a. $3(n + 2)$ b. $n + 2$ c. $\frac{n+2}{3}$ d. $3n + 2$	Todd spends $3 of his monthly allowance on crayons and then spends one-half of his remaining money on activity books. If he has $6.50 left, what was the amount of his monthly allowance? Solve this problem by writing an equation with a variable and then solve the equation. Be sure to specify what the variable in your equation represents.	You ask your students to model and solve the equation $3x = 15$. Describe or draw a model appropriate for this equation. Explain how your model can be used to solve the equation.

*The graphs are omitted because of space considerations.

Appendix C

Sample Items for Diagnostic Teacher Assessments for Mathematics and Science (DTAMS), Rational Numbers Assessment.

Subcategory	Type 1—memorized/ factual knowledge	Type 2—conceptual understanding	Type 3—reasoning/problem solving	Type 4—pedagogical content
Elementary Number and Integer Concepts and Representations	Which of the following fractions in ***NOT*** equivalent to 1/6? a. 2/12 b. 3/18 c. 4/24 d. 5/36	What is the relationship between the numerator and denominator in any fraction that is equivalent to 1/10? a. The numerator is 10 times the denominator b. The denominator is 10 times the numerator c. The numerator is 10 more than the denominator d. The denominator is 10 more than the numerator	Drake and Lesley decided to use a piece of construction paper to build paper airplanes. Drake's piece of paper measured 7 in. by 9 in. Lesley's piece of paper measured 6 in. by 12 in. Drake used 2/3 of his paper to build his plane and Lesley used ¾ of her paper to build her plane. a. Compare the two airplanes with respect to overall area. b. Show me your work.	Two years ago, the population of Largeville was 600. If it increased in population by 10% each year for two years, what is its population <u>now</u>? Kathy solved this problem by noting that because 10% of 600 is 60, the new population should be 600 + 60 + 60 = 720 people. a. Is Kathy correct? Why or why not? b. If not, describe an instructional activity that will support her understanding and correct her misconception.

Operations/ Computation	What is: $4 - 2\frac{1}{6}$? a. 5/12 b. 2 c. 1 5/12 d. 1 5/6	Jenny bought a sandwich for $1.65 and a drink for $0.80. Which of these shows the way to find how much change Jenny received from $5.00? a. Add $0.80, $1.65, and $5.00 b. Add $1.65 and $0.80 and then subtract the total from $5.00 c. Add $1.65 and $5.00 and then subtract $0.80 from the total d. Add $0.80 and $5.00 and then subtract $1.65	The student population at Ben Franklin School is growing. Two years ago there were 500 students. Last year the student population grew by 1/10. This year the population grew 1/10 of the previous year. a. How many students does Ben Franklin School have this year? b. Does an increase in population by 1/10 two years in a row equal a population growth of 1/5? Explain your reasoning.	Tom said, “I can multiply 2/3 by some other number and get a. A number less than 2/3.” [Explain why this is always possible.] b. A number less than zero.” [Explain why this is always possible.]

Appendix D

"The Monroe Doctrines"

1. Always understand the content beyond what you need to teach to your students before you teach.
2. Affirm your students as needed by restating what they have said and then build upon it by "and so . . . ."
3. Know that manipulatives are tools. Students need to select the tool they need to do their work of problem solving.
4. Scaffold students as much as they need—and no more. Too little scaffolding promotes confusion; too little scaffolding enables dependency.
5. Teach procedures and formulas after students understand the concept.
6. Always have at least two different ways to solve the problem; your students should, too.
7. Remember equity—all children can learn, all children have worth, and all have ideas worth sharing.
9. Teaching and learning in a discourse community demands collaboration.
7. To be meaningful, mathematics must be learned "from the inside out."
10. Remember that children's thinking is at the heart of teaching and learning.

Footnote

[1]*Puka* is the Hawaiian word, at the same time both singular and plural, for "hole." It originated with the naturally occurring hole in a puka shell. (*Nā* preceding the noun *puka* makes it plural.) The authors thank EileenTanaka, one of the project participants, for reminding the participants, "Oh, we're filling in the pukas [*nā puka*] in our understanding!"

Table 1

IMAP Pretest and Posttest Scores According to Participant

Participant	Total pretest score	Total posttest core	Significance
1	17	17	
2	24	30	
3	7	12	
4	25	26	
5	17	29	
6	19	21	
7	19	23	
8	16	28	
9	25	30	
10	23	31	
11	10	18	
12	18	24	
13	17	16	
14	17	23	
15	25	31	
16	11	11	
17	9	13	
18	18	21	
19	22	27	

20	25	26	
21	16	24	
22	14	15	
23	12	14	
24	10	15	
$\bar{X}$	17.3	21.8	$t(23) = 2.60$ ($p < .01$)
SD	5.5	6.5	

Table 2

IMAP Pretest and Posttest Scores According to Belief

IMAP belief	Total pretest score	Total posttest score	Significance ($p < .05$)
1	23	45	$t\ (23) = 3.01$ ($p < .01$)
2	46	40	Not significant
3	67	90	t (23) = 1.87 ($p < .05$)
4	65	81	Not significant
5	96	97	Not significant
6	62	98	t (23) = 3.43 ($p < .01$)
7	58	74	t (23) = 1.88 (p < .05)

Table 3.

Results of Paired Student's t-test for DTAMS Probability/Statistics/Algebra Pretest and Posttest Scores (, **, and *** indicate significance at p < .05)*

Subject	**Probability***		**Data Representation and Analysis****		**Algebraic Ideas**		**Overall*****	
	Pre	**Post**	**Pre**	**Post**	**Pre**	**Post**	**Pre**	**Post**
1	4	8	8	9	7	6	19	23
2	6	6	4	6	11	6	21	18
3	2	6	8	7	5	7	15	20
4	2	9	4	5	8	6	14	20
5	2	3	4	6	9	6	15	15
6	4	7	10	10	4	7	18	24
7	2	3	6	7	6	9	14	19
8	2	8	8	11	8	6	18	25
9	2	6	5	9	9	5	16	20
10	5	9	8	12	9	10	22	31

*p < 0.0009

** p < 0.0095

*** p < 0.00

Table 4.

Results of Paired Student's t-test for DTAMS Rational Numbers Pretest and Posttest Scores (no significant differences [$p < .05$])

Subject	**Operations/Computation**		**Concepts and Representations**		**Overall**	
	Pre	**Post**	**Pre**	**Post**	**Pre**	**Post**
1	4	5	16	18	20	23
2	7	8	17	19	24	27
3	11	9	20	20	31	29
4	4	4	7	6	11	10
6	6	11	17	24	23	35
7	7	12	17	20	24	32
8	3	5	17	22	20	27
10	13	12	25	25	38	37
11	8	8	18	25	26	33

.